PRICING MANUAL OPTIONS CONTRACTS

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CONTENTS

INTRODUCTION

In this Manual, we present the methodologies for calculating the reference option premiums and the necessary inputs, such as implied volatilities.

If the inputs described in this document -total or partially- were unavailable (consequently the methodology described cannot be applied), B3 could arbitrarily set the prices by means of Risk Committee approvement or use the same volatility of the day before.

1 EQUITIES

1.1 Equity, ETF and index options contracts

The reference premium for call and put options is calculated according to equations (1.1) and (1.2), respectively:

$$PRCALL_n = S \times e^{(-q_n T_n)} \times N(d_1) - K \times e^{(-r_n T_n)} \times N(d_2) \quad (1.1)$$

$$PRPUT_n = -S \times e^{(-q_n T_n)} \times N(-d_1) + K \times e^{(-r_n T_n)} \times N(-d_2) \quad (1.2)$$

where:

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r_n - q_n + \frac{\sigma^2}{2}\right) T_n}{\sigma\sqrt{T_n}} \quad (1.3)$$

$$d_2 = \frac{\ln\left(\frac{S}{K}\right) + \left(r_n - q_n - \frac{\sigma^2}{2}\right) T_n}{\sigma\sqrt{T_n}} \quad (1.4)$$

S is the closing price of the option's underlying instrument;

r_n is the exponential interest rate in continuous regime and on annual basis related to the n contract month and calculated according to equation (1.5) below;

q_n is the exponential carrying cost (or convenience yield) in continuous regime and on an annual basis related to the n contract month and calculated according to equation (1.6) below;

T_n is the contract month term in calendar years pertaining to the marketplace in question, namely:

$$T_n = \frac{DU_n}{252}$$

where DU_n is the number of withdrawal days between the calculation date and contract month date of the i interpolated contract month;

K is the option's exercise price; and

σ is the option's volatility calculated according to section 1.2.

Calculation of the exponential interest rate

$$r_n = \ln(1 + TPre_{DI1}^n) \quad (1.5)$$

where:

$TPre_{DI1}^n$ is the prefixed rate for the n contract month calculated according to the exponential interpolation of the futures contract settlement price with a one-day ID (ID1) average rate (see the BM&FBOVESPA PRICING MANUAL – FUTURES CONTRACTS).

Calculation of the exponential carrying cost (or convenience yield) rate

$$r_n = \ln(1 + TCY_n) \quad (1.6)$$

where:

TCY_n is the carrying cost (or convenience yield) rate for the maturity n calculated according to the futures contract settlement price of the underlying asset, the interest rate and interpolated by maturity.

Note: The carrying cost (or convenience yield) rate is zero for all options of stocks, ETFs and Indices, except for options on the Ibovespa Index, where the rate is extracted from the Ibovespa Futures Contract (IND), as described in Section 5.4 of the B3 Curves Manual.

1.2 Calculation of volatility for equity, ETF and index options

The volatility for equity, ETF and index options will be calculated according to two family models depending on the liquidity of the options series. The procedures summarized below are applied by underlying instrument.

Calculation models for liquid options

These models are applied to generate volatility surfaces for equity, ETFs and indices that have a minimum series (see Table 1 of the Monthly Parameters Annex) with liquidity.

Liquidity assessments of the series for use of these models are done every two weeks. The list of assets classified as liquid is shown in Table 1 of the Monthly Parameters Annex. Such classification does not establish minimum quantities or spreads. As will be further explained, quantities and spreads are considered when adjusting the volatility surface models to the trades and orders observed.

The generation of the volatility surface for all options series of an asset (equity, ETF or index) classified as liquid is done in two steps:

1. Liquid series: trades and orders verified in the capture window (see Table 1 of the Monthly Parameters Annex) that precedes the closing of the options trading are used to adjust the non-arbitrage models to volatility surfaces; and

2. Illiquid series: volatility is obtained from the adjusted models in the previous step.

This approach ensures the generation of arbitrage-free volatilities and premiums.

The previous calculation steps are performed for call and put options separately, i.e., different volatility surfaces are produced for call and put options.

Calculation model for illiquid options

This model applies to the generation of volatility surfaces related to assets classified as illiquid, namely, those which do not contain the minimum number of liquid series.

The same models are used for equity regarded as liquid, but the parameter adjustment of the model is done from the equity historical data.

This approach ensures the development of volatility surfaces with the same characteristics observed in the liquid series, such as volatility smile and volatility term structure, as well as ensuring the generation of volatilities and arbitrage-free premiums.

1.2.1 Illiquid calculation model

For equity, ETFs and indices classified as illiquid (assets not listed in Table 1 of the Monthly Parameters Annex), the volatility surface is calculated by following the steps below, which are applied to both call and put options.

Step 1: Capture of closing data: asset closing price and risk-free interest rate curve (see BM&FBOVESPA PRICING MANUAL – FINANCIAL ASSETS FUTURES CONTRACTS)

Closing prices update the price data history used in calculating logarithmic returns.

Step 2: Calculation of higher order sample moments: asymmetry and kurtosis (item 1.2.1.2)

The higher order sample moments are calculated according to the three-year history of logarithmic returns for closing prices.

Step 3: Calculation of the volatility term structure (item 1.2.1.3)

Contract month volatilities corresponding to options contract months are calculated from the GARCH model (1.1). The instantaneous volatility of the GARCH model (1.1) is updated daily, while ω, α, β coefficients and long-term volatility are updated weekly.

Step 4: Calculation of options premiums

The premiums are calculated based on the Corrado & Su model, using the volatilities in the volatility term structure and the sample moments (item 1.2.1.1).

Step 5: Calculation of implied options volatilities

The implied volatilities of all options series are calculated according to the Corrado & Su premiums (whose calculation was performed in the previous step) by reversing the Black-Scholes equation (subsection 1.2.2.8).

1.2.1.1 Corrado & Su model

Call options

The Corrado & Su model calculates the options premiums. The value of a European call option is given by:

$$C_{CS}(S,K,r,q,T,\sigma,\kappa_3,\kappa_4) = C_{BS}(S,K,r,q,T,\sigma) + \kappa_3 Q_3 + (\kappa_4 - 3)Q_4$$

where:

$$Q_3 = \frac{1}{6(1+w)} S\sigma\sqrt{T}\left(2\sigma\sqrt{T} - d\right)n(d)$$

$$Q_4 = \frac{1}{24(1+w)} S\sigma\sqrt{T}\left(d^2 - 3d\sigma\sqrt{T} + 3\sigma^2 T - 1\right)n(d)$$

with:

$$d = \frac{\ln\left(\frac{S}{K}\right) + \left(r - q + \frac{\sigma^2}{2}\right)T - \ln(1+w)}{\sigma\sqrt{T}}$$

$$w = \frac{\kappa_3}{6}\sigma^3 T^{3/2} + \frac{\kappa_4}{24}\sigma^4 T^2$$

$C_{BS}(S,K,r,q,T,\sigma)$ is the premium of a call option using the Black-Scholes model;

S is the price of the underlying instrument (closing price);

K is the option's exercise price;

r is the exponential interest rate in continuous regime and on an annual basis;

q is the exponential carrying cost (or convenience yield) in continuous regime and on an annual basis related to the n contract month and calculated according to equation (1.6);

T is the contract month term in calendar years pertaining to the marketplace in question;

σ is the model's volatility obtained from the volatility term structure (item 1.2.1.3); and

κ_3 and κ_4 is the asymmetry and kurtosis of the underlying instrument (item 1.2.1.2).

Put options

The put options price according to the Corrado & Su model is determined by the put-call parity:

$$P_{CS}(S, K, r, q, T, \sigma, \kappa_3, \kappa_4) = C_{CS} - S\exp(-qT) + K\exp(-rT)$$

with $C_{CS} \equiv C_{CS}(S, K, r, q, T, \sigma, \kappa_3, \kappa_4)$, which is the call option price in the Corrado & Su model for the same exercise prices and contract month.

1.2.1.2 Calculation of higher order moments

The higher order moments are calculated according to the following equations:

$$\kappa_3 = \sum_{j=t}^{t-N} \frac{1}{N} \frac{(r(j)-m)^3}{s^3} \text{ and } \kappa_4 = \sum_{j=t}^{t-N} \frac{1}{N} \frac{(r(j)-m)^4}{s^4}$$

where:

$r(j) = \ln(S_j / S_{j-1})$ are the logarithmic returns;

m and s are the median and standard deviation of returns; and

N is the size of the historical data used in the calculations (in this case, $N = 3$ years of daily returns).

1.2.1.3 Calculation of the volatility term structure

The volatility parameter σ in the Corrado & Su model is a function of the option's contract month term, $\sigma \equiv \sigma(T)$, which is the volatility term structure:

$$\sigma(T) = \sqrt{252\, V(T)}$$

$$V(T) = V_L + \frac{1 - \exp(-aT \cdot 252)}{aT \cdot 252}\left(\hat{\sigma}^2(t+1) - V_L\right)$$

with:

$$a = \ln \frac{1}{\alpha + \beta}$$

$$V_L = \frac{\omega}{1 - \alpha - \beta}$$

where:

T is the term corresponding to the option's contract month on business days;

α, β and ω are the GARCH(1,1) model's coefficients;

$\hat{\sigma}^2(t+1)$ is the instantaneous variance calculated according to the autoregressive volatility formula of the GARCH model(1,1).

$$\hat{\sigma}^2(t+1) = \omega + \alpha r^2(t) + \beta \hat{\sigma}^2(t)$$

with:

$r(t)$ is the last instant of the series of returns (calculated at the day's closing);

$\hat{\sigma}^2(t)$ is the autoregressive variance estimator obtained from the application of the formula above the series of returns and considering the sample variance as the variance at the origin $\hat{\sigma}^2(t-N-1)$ for a series of N length returns.

1.2.2 Liquid calculation model

For equity, ETFs and indices classified as liquid and contained in Table 1 of the Monthly Parameters Annex, the volatility surface is calculated according to the following steps, which are applied to call and put options separately.

Step 1: Capturing intraday trading data and calculating the average price and its uncertainty for each series (item 1.2.2.6.1)

Data are captured in the capture window (see Table 1 of the Monthly Parameters Annex) that precedes the closing of the trading session for each series:

- Trades (quantity and price) carried out; and

- Call and put orders (quantity and price). The orders available in the first level of the order book offering simultaneous bid and ask prices are considered. Each change in price or quantity results in a new entry.

Step 2: Calculating the implied volatility regarding average prices of series and their uncertainties (item 1.2.2.6.2)

The following calculations apply for each series:

- the implied volatility of the series' median prices; and

- the uncertainties of each implied volatility based on the higher and lower limits defined by the uncertainty over the average price of the underlying instrument

Step 3: Adjustment of non-arbitrage model

The non-arbitrage models for the series of options are adjusted through the average prices of the underlying instruments, series and implied volatilities (item 1.2.2.1). However, before defining the non-arbitrage model, it is necessary to classify the contract months. Contract months with a number of series above the minimum quantity (see Table 1 of the Monthly Parameters Annex) are classified as liquid contract months; other contract months that do not meet this criterion are classified as illiquid. Given the classification of contract months, the adjustment of the models can be done in two ways:

- directly on liquid contract months; or

- directly on liquid and illiquid contract month clustering.

Adjustment of liquid contract months

Liquid contract months can be adjusted using two models:

- SABR implied volatility model: the adjustment considers the median implied volatilities and their uncertainties; and

- Corrado & Su options premium model: the adjustment considers the median premiums and their uncertainties.

The models are applied to the equity and are available in Table 1 of the Monthly Parameters Annex. Eventually, the models can be changed, which occurs when an alternate model exhibits better adjustment than the standard model defined for the equity.

Evaluation of the quality of liquid model adjustment

The quality of the model adjustment is assessed based on the distribution of the model's residual, both with regard to the observed premiums and to the observed implied volatilities. Errors in the model adjustment must be covered by the uncertainties associated with the series. Eventually, for some market movements, the observed curves may hinder the convergence of the adjustment, thus generating results where the uncertainty of the observed data is higher than the model's residual. This result is classified as a violation. When some series present violations, alternative models must be tried in order to reduce such violations.

Adjustment of liquid and illiquid contract month clustering

Contract month clustering can be adjusted by means of two models: VLFit and VLGARCH, which consider the average premiums and their uncertainties, as detailed in items 1.2.2.6 and 1.2.2.7.

These models are applied to equity and are available in Table 1 of the Monthly Parameters Annex. Eventually, the models can be changed and this occurs when an alternative model presents a better adjustment than the standard model defined for the equity.

Step 4: Calculation of the options' implied volatilities

Implied volatilities of all options series for each equity, ETF and index classified as liquid and calculated in the previous step according to the Corrado & Su, VLFit and VLGARCH models are obtained by inversion of the Black-Scholes equation. The other implied volatilities are estimated through the SABR model (subsection 1.2.2.8).

1.2.2.1 Adjustment of non-arbitrage models

Non-arbitrage models are adjusted by minimizing the fitness function:

$$f_{obj} = \sum_{i=1}^{N} \left(\frac{f_i - y_i}{\sigma_{y_i}} \right)^2$$

where:

N is the quantity of series with data capture information;

f_i is the function of the model adopted in the adjustment;

y_i is the median of captured data – premiums or implied volatilities; and

σ_{y_i} is the uncertainty related to y_i.

In sections 1.2.2.2 to 1.2.2.5, the functions of the models f_i used in the adjustments are shown. The optimizer used is an implementation of the Globally-Convergent Method of Moving Asymptodes (MMA) (described in Krister Svanberg as "a class of globally convergent optimization methods based on conservative convex separable approximations," SIAM J. Optim. 12 (2) p. 555-573 (2002)).

In section 1.2.2.6 the expressions for calculating the medians and uncertainties of premiums and implied volatilities are presented.

1.2.2.2 SABR model

SABR is an implied volatility model:

$$\sigma_{BS}(F,K,T) = A_1.\left(\frac{z}{x(z)}\right).[1 + A_2.T]$$

where:

$$A_1 = \frac{\alpha}{(FK)^{(1-\beta)/2}\left\{1 + \frac{(1-\beta)^2}{24}\left[\ln\left(\frac{F}{K}\right)\right]^2 + \frac{(1-\beta)^4}{1920}\left[\ln\left(\frac{F}{K}\right)\right]^4\right\}}$$

$$A_2 = \frac{(1-\beta)^2}{24}\frac{\alpha^2}{(FK)^{1-\beta}} + \frac{1}{4}\frac{\rho\beta\nu\alpha}{(FK)^{(1-\beta)/2}} + \frac{2-3\rho^2}{24}\nu^2$$

$$z = \frac{\nu}{\alpha}(FK)^{(1-\beta)/2}\ln(F/K)$$

$$x(z) = \ln\left\{\frac{\sqrt{1-\rho z + z^2} + z - \rho}{1-\rho}\right\}$$

with:

S and $F = S\frac{e^{rt}}{e^{qt}}$is the price of the underlying instrument and its future value. The value of the underlying instrument is calculated according to item 1.2.2.6;

K is the exercise price;

T is the annual term for the option contract month;

r is the exponential interest rate in continuous regime and on an annual basis;

q is the exponential carrying cost (or convenience yield) in continuous regime and on an annual basis related to the n contract month and calculated according to equation (1.6); and

α, β, ρ and ν are the parameters adjusted to the captured data related to the liquid contract months. Such data are calculated according to item 1.2.2.6.

1.2.2.3 Corrado & Su model

The Corrado & Su model (item 1.2.1.1) is given by:

$$C_{CS}(S, K, r, q, T, \sigma, \kappa_3, \kappa_4)$$

where:

S is the price of the underlying instrument calculated according to item 0;

K is the exercise price;

T is the annual term for the option contract month;

r is the exponential interest rate in continuous regime and on an annual basis;

q is the exponential carrying cost (or convenience yield) in continuous regime and on an annual basis related to the n contract month and calculated according to equation (1.6); and

σ, κ_3 and κ_4 are the parameters adjusted to the captured data related to the liquid contract months. Such data are calculated according to item 1.2.2.6.

1.2.2.4 VLGARCH model

The Corrado & Su model (item 0) is given by:

$$C_{CS}(S, K, r, q, T, \sigma, \kappa_3, \kappa_4)$$

where:

S is the price of the underlying instrument calculated according to item 0;

K is the exercise price;

T is the annual term for the option contract month;

r is the exponential interest rate in continuous regime and on annual basis;

q is the exponential carrying cost (or convenience yield) in continuous regime and on annual basis related to the n contract month and calculated according to equation (1.6); and

σ, κ_3 and κ_4 are the parameters adjusted to the captured data related to the liquid contract months. Such data are calculated according to item 0.

$\hat{\sigma}^2(t+1)$, a, κ_3 and κ_4 are the parameters adjusted to the captured data related to the liquid contract months. Such data are calculated according to item 0;

$\sigma \equiv \sigma(T; a, \hat{\sigma}^2(t+1), V_L)$ is given by the volatility term structure (item 1.2.1.3); and

V_L is the same long-term volatility used in the illiquid model (item 1.2.1.3) and calculated according to the GARCH parameters of the equity.

1.2.2.5 VLFit model

The Corrado & Su model (item 0) is given by:

$$C_{CS}(S, K, r, q, T, \sigma, \kappa_3, \kappa_4)$$

S is the price of the underlying instrument calculated according to item 0;

K is the exercise price;

T is the annual term for the option contract month;

r is the exponential interest rate in continuous regime and on an annual basis;

q is the exponential carrying cost (or convenience yield) in continuous regime and on an annual basis related to the n contract month and calculated according to equation (1.6);

$\sigma \equiv \sigma(T; a, \hat{\sigma}^2(t+1), V_L)$ is given by the volatility term structure (item 1.2.1.3); and

V_L, $\hat{\sigma}^2(t+1)$, a, κ_3 e κ_4 are the parameters adjusted to the captured data related to the liquid contract months. Such data are calculated according to item 0.

1.2.2.6 Consolidation of equity options intraday data

According to item 1.2.2.1, non-arbitrage models are used to adjust (i) observed premiums and (ii) implied volatilities in the observed premiums, whereas the adjustment is made from the median values and the uncertainties associated with each series observed.

The median values and the uncertainties of each series are calculated based on the observations of:

- equity transactions;
- options transactions;
- options call and put orders.

The uncertainties are calculated from the trades and orders observed during the capture period. The capture period is set for the last ten minutes prior to the closing auction on the spot stock market.

The calculations of the options' and implied volatilities' median values and uncertainties are further demonstrated below.

1.2.2.6.1. Options' median values and uncertainties

Calculation of the options median price

The options' median price is calculated in three steps as shown below.

1. Calculation of the options transaction median price

$$p_n = \frac{\sum_{i=1}^{N} Q_i P_i}{\sum_{i=1}^{N} Q_i}$$

where:

Q_i is the quantity of options traded in the ith transaction during the capture period;

P_i is the price corresponding to the ith options transaction during the capture period; and

N is the quantity of options transactions carried out during the capture period.

2. Calculation of the median price of options orders (mid-price)

$$p_{mid} = \frac{p_c + p_v}{2}$$

where:

p_c and p_v are the median call and put order prices, respectively.

$$p_X = \frac{\sum_{i=1}^{N} Q_{X,i} P_{X,i}}{\sum_{i=1}^{N} Q_{X,i}}$$

where:

p_X is the median call order price ($X = c$) or put order price ($X = v$);

$Q_{X,i}$ is the quantity of contracts offered (in X, call or put) in the ith order observed at the top of the order book during the capture period;

$P_{X,i}$ is the price corresponding to the ith order (in X, call or put) observed during the capture period; and

N is the quantity of orders observed during the capture period.

3. Composition of median orders with median transactions in the final median options prices

$$p_{opt} = \frac{\frac{p_n}{s_n^2} + \frac{p_{mid}}{s_{mid}^2}}{\frac{1}{s_n^2} + \frac{1}{s_{mid}^2}}$$

where:

s_n and s_{mid} are uncertainties related to the median transaction price and the median order price, respectively. The calculation of these variables is presented below.

Calculation of the option price uncertainty

The uncertainty of the options' median price is calculated in three steps as follows.

1. Calculation of transaction uncertainty

$$s_n = \frac{\sigma_n \sqrt{\sum_{i=1}^{N} Q_i^2}}{\sum_{i=1}^{N} Q_i}$$

with:

$$\sigma_n = \sqrt{\frac{\sum_{i=1}^{N} (P_i - P_n)^2}{N - 1}}$$

where:

Q_i is the quantity of options traded in the ith transaction during the capture period;

P_i is the price corresponding to the ith options transaction during the capture period; and

N is the quantity of options transactions carried out during the capture period.

When $N \leq 1$, we assume $s_n = 0{,}005$ (the uncertainty in the price is half a cent of Brazilian Reals). These parameters can be specified per equity and are listed in Table 2 of the Monthly Parameters Annex.

A correction is applied to uncertainty to avoid distortions when there are few (typically less than five) large volume transactions during capture. Correction is performed by multiplying the f_t factor by s_n:

$$s_n \equiv f_t \, s_n$$

where:

$$f_t = \frac{q(IC, N-1)}{q(IC, \infty)}$$

with:

$q(IC, \nu)$ is the inverse function of the t-student distribution with ν degrees of freedom and IC confidence interval. The IC parameter is defined in Table 2 of the Monthly Parameters Annex.

When ≤ 1 , we use:

$$f_t = \frac{q(IC, 1)}{q(IC, \infty)}$$

2. Calculation of order price uncertainty

$$s_{mid} = \sqrt{\frac{1}{4}(s_c^2 + s_v^2) + \left(\frac{spread}{2}\right)^2}$$

where:

s_{mid} is the mid-price uncertainty (median of call and put orders);

s_c, s_v is the call and put order uncertainty; and

$spread$ is the difference between the median of the call and put prices offered, namely:

$$spread = p_v - p_c$$

$$s_X = \frac{\sigma_X \sqrt{\sum_{i=1}^{N} Q_{X,i}^2}}{\sum_{i=1}^{N} Q_{X,i}}$$

with:

$$\sigma_X = \sqrt{\frac{\sum_{i=1}^{N} \left(P_{X,i} - p_X\right)^2}{N-1}}$$

where:

s_X is the uncertainty of the median call price ($X = c$) or put price ($X = v$);

p_X is the median call price ($X = c$) ou put price ($X = v$);

$Q_{X,i}$ is the quantity of contracts offered (in X, call or put) in the ith order observed in the order book during the capture period;

$P_{X,i}$ is the price corresponding to the ith order (in X, call or put) observed during the capture period;

N is the quantity of orders observed during the capture period; and

3. Composition of order uncertainties with options' transaction uncertainties

The final uncertainty of options price is given by:

$$s_{opt} = \sqrt{\frac{1}{\frac{1}{s_n^2} + \frac{1}{s_{mid}^2}}}$$

Adjustment of uncertainty by quantities of trades and orders

The quantities of trades and orders are directly linked to options price uncertainties, since the different levels of these quantities determine the quality of pricing. The purpose here is to redistribute the series weights at each contract month separately according to the volume of trades and orders, as well as the number of transactions and updates in the first level of the order book. Therefore, given the weights (uncertainties) obtained through price swings and offer spreads, we wanted to include a portion of the weight due to the number of transactions and trading volume.

For this reason, a portion of the weight was included in s_{opt} (final uncertainty of the options price):

$$s_{opt} \equiv \sqrt{\alpha\left(s_{opt}\right)^2 + (1-\alpha)\left(s_q\right)^2}$$

where:

α is the weight attributed to the portion related to the final uncertainty of the options price limited to the $0 \leq \alpha \leq 1$ interval. This parameter is defined in Table 2 of the Monthly Parameters Annex; and

s_q is the uncertainty associated with the quantity of trades, quantity of orders and the number of contracts traded and offered. This amount relates to the contract month of the series in question, so as to correct the final uncertainty by effect of the quantities of trades and orders.

The s_q calculation involves the quantities of trades, the quantities of offers and the number of contracts traded and offered. The steps and the formulas will be broken down to arrive at the s_q. To simplify the nomenclature, the quantities of trades and orders are named events since they represent the observed events. The following calculations are performed by contract month (*smile*). Therefore, N is considered the number of series with information at the contract month and M is the number of trades (or orders) in the *i*th series.

1. Calculation of the number of trades and orders (events)

The number of final events for each i series in a contract month is given by:

$$n_i = f_n \alpha_n n_i^{neg} + (1 - \alpha_n) n_i^{of}$$

with:

$\alpha_n = 0 \leq \alpha_n \leq 1$ is the factor that defines the weight to be given to the number of trades versus the number of order events (see Table 2 of the Monthly Parameters Annex);

$$n_i^{neg} = \sum_{j=1}^{M} neg_j$$

with:

neg_j are the trades observed in the jth event of the i series;

$$n_i^{of} = \sum_{j=1}^{M} of_j$$

where:

of_j are the orders observed in the jth event of the i series; and

f_n is the normalization factor between orders and trades (by contract month), namely:

$$f_n = \frac{\sum_{i=1}^{N} n_i^{of}}{\sum_{i=1}^{N} n_i^{neg}}$$

2. Calculation of the number of trade contracts and order contracts

The number of final contracts for each i series of a contract month is given by:

$$q_i = f_q . \alpha_q . q_i^{neg} + \left(1 - \alpha_q\right) . q_i^{of}$$

where:

α_q: $0 \leq \alpha_q \leq 1$ is the factor that defines the weight to be given to the number of contracts traded versus the number of contracts offered (see Table 2 of the Monthly Parameters Annex);

$$q_i^{neg} = \sum_{j=1}^{M} q_j^n$$

with:

q_j^n is the number of contracts traded in the jth event of the i series;

$$q_i^{of} = \sum_{j=1}^{M} q_j^o$$

where:

q_j^o is the number of contracts offered in the jth event of the i series; and

f_q is the normalization factor between orders and trades (by contract month), namely:

$$f_q = \frac{\sum_{i=1}^{N} q_i^{of}}{\sum_{i=1}^{N} q_i^{neg}}$$

3. Normalization between contracts and events

For each i series, the number of events and the volume of contracts are normalized through the following equation:

$$q_i^{nq} = f_{nq}.\alpha_{nq}.n_i + (1 - \alpha_{nq}).q_i$$

thus generating the final q_i^{nq} quantity related to the i series, which integrates the number and size of events, where:

q_i^{nq} is the normalized quantity considering the number of events and quantity of trades;

α_{nq} = $0 \leq \alpha_{nq} \leq 1$ is the factor that regulates the number of events and the volume of contracts (see Table 2 of the Monthly Parameters Annex);

f_{nq} is the scale factor between the number of events and the volume of contracts, namely:

$$f_{nq} = \frac{\sum_{i=1}^{N} q_i}{\sum_{i=1}^{N} n_i}$$

4. Calculation of the uncertainty associated with the s^q quantity of trades and orders

The uncertainty associated with the quantity of trades and orders corresponding to the i series is given by:

$$s_q \equiv s_i^q = \frac{1}{w_i^q}$$

where:

$$w_i^q = \frac{\sum_{i=1}^{N} \frac{1}{s_i^{opt}}}{\sum_{i=1}^{N} q_i^{nq}} \; q_i^{nq}$$

with:

s_i^{opt} is the options' price uncertainty related to the i series. It should be noted that, according to what was previously described, $s_{opt} \equiv s_i^{opt}$ is the uncertainty of the options price related to the i series. In analogous manner, $s_q \equiv s_i^q$. Therefore, we have:

$$s_{opt} \equiv \sqrt{\alpha\left(s_{opt}\right)^2 + (1-\alpha)\left(s_q\right)^2}$$

which is the final options' price uncertainty. This uncertainty is used for the calculation of the implied volatility of the i series option, as will be further addressed below.

1.2.2.6.2. Median values and uncertainties for implied volatilities

Calculation of the median value of implied volatility

The median value of implied volatility is calculated according to the following formula:

$$V = \sigma_{BS}\left(p_{opt}, p_a, K, r, q, T\right)$$

where:

p_{opt} is the final median of the options' price;

p_a is the median price of the underlying instrument;

K is the exercise price;

T is the option's contract month term;

r is the exponential interest rate in continuous regime and on an annual basis;

q is the exponential carrying cost (or convenience yield) in continuous regime and on an annual basis related to the n contract month and calculated according to equation (1.6); and

$\sigma_{BS(\ldots)}$ is the calculation of the implied volatility (subsection 1.2.2.8)

Calculation of the implied volatility uncertainty from price uncertainty

The implied volatility uncertainty of the i series is given by:

$$s_V = \frac{|V_u - V_d|}{2}$$

where:

$$V_u = \sigma_{BS}\left(p_{opt} + s'_{opt}, p_a, K, r, q, T\right)$$

$$V_d = \sigma_{BS}\left(p_{opt} - s'_{opt}, p_a, K, r, q, T\right)$$

with:

p_{opt} is the final median of the options' price;

s'_{opt} is the final options' price uncertainty, namely:

$$s'_{opt} = \sqrt{s_{opt}^2 + (\Delta \cdot \sigma_a)^2}$$

where:

Δ is the option's delta (Δ_{CALL} for call options and Δ_{PUT} for put options), with:

$$\Delta_{CALL} = N(d_1) \text{ and } \Delta_{PUT} = N(d_1) - 1$$

σ_a is the equity price uncertainty;

p_a is the median price of the underlying instrument;

K is the exercise price;

T is the option's contract month term;

r is the exponential interest rate in continuous regime and on annual basis;

q is the exponential carrying cost (or convenience yield) in continuous regime and on an annual basis related to the n contract month and calculated according to equation (1.6); and

$\sigma_{BS(\ldots)}$ is the calculation of the implied volatility (subsection1.2.2.8)

Calculation of the equity median price

The equity median price is given by:

$$p_a = \frac{\sum_{i=1}^{N} q_i p_i}{\sum_{i=1}^{N} q_i}$$

where:

q_i is the volume of equities traded in the ith transaction during the capture period;

p_i is the price corresponding to the ith transaction during the capture period; and

N is the quantity of trades carried out during the capture period.

Calculation of the equity price uncertainty

The equity price uncertainty is given by:

$$s_a = \frac{\sigma_a \sqrt{\sum_{i=1}^{N} q_i^2}}{\sum_{i=1}^{N} q_i}$$

with:

$$\sigma_a = \sqrt{\frac{\sum_{i=1}^{N} (p_i - p_a)^2}{N - 1}}$$

1.2.2.7 Options clustering for volatility model adjustment

Before being adjusted, the series undergo a selection considering the following points:

- series with absolute delta value below the maximum delta;
- series with absolute delta value above the minimum delta; and
- series with uncertainty smaller than the maximum uncertainty.

Following the selection, the series are clustered together to adjust the volatility models. There are two scenarios for adjusting the models:

1. Model adjustment by contract month: the options series on the same underlying instrument and of the same type (call or put) are clustered by contract month. Contract months that contain the minimum quantity (defined in Table 2 of the Monthly Parameters Annex) are adjusted by the contract month model defined in Table 1 of the Monthly Parameters Annex. The Corrado & Su and SABR models are alternatives for adjustment at the contract month; and

2. Model adjustment by options block: the options series on the same underlying instrument and of the same type (call or put) are clustered by blocks, so that it is possible to adjust a model with data from different contract months. The VLGARCH and VLFit models are alternatives for adjustment by options block. The steps for building the blocks are shown below.

Steps for building the options blocks

The creation of the blocks is applied to options on the same underlying instrument and of the same type (call or put).

Step 1: Counting and identifying pivotal contract months: pivotal contract months follow the minimum quantity of series per contract month. Other series which do not belong to pivotal contract months are illiquid and, therefore, the contract months of these series are illiquid contract months. It should be noted that such series and contract months are illiquid on an asset that is classified as liquid.

Step 2: Each group of illiquid series and contract months may be associated with up to two pivotal contract months. There are four possible scenarios:

1. Illiquid contract months from the beginning of the term structure associated with a later pivotal contract month;

2. Illiquid intermediary contract months of the term structure associated with an earlier and later pivotal contract month;

3. Illiquid contract months at the end of the term structure associated with an earlier pivotal contract month; e

4. Absence of pivotal contract months leading to the creation of a single options block.

Necessary conditions

- Illiquid contract months can only be part of a single block.
- Pivotal contract months can only be part of two blocks when they belong to the interface between the blocks.
- The minimum number of series per block must follow the minimum number of series required for each surface model.

Sufficient condition

- Two pivotal contract months ensure the minimum number of series required to optimize any of the adopted models (VLGARCH or VLFit).

1.2.2.8 Calculation of implied volatility in the Black-Scholes model

The calculation of the implied volatility by the Black-Scholes formula is done through an iterative process that seeks to find the σ value, which is the root of the following equation

$$BS(S, K, r, q, T, \sigma) - premium = 0$$

where:

BS is the Black-Scholes model (section 1.1); and

$premium$ is the reference premium.

The remaining parameters S, K, r, q, T are the same parameters used in the calculation of the options' premium.

With the purpose of simplifying the notation, the following function is considered:

$$\sigma_{BS}(premium; S, K, r, q, T)$$

representing the solution of the iterative process that solves the above equation. The bisection or Newton-Raphson methods are indicated for implementation of the iterative process.

2 CURRENCIES

2.1 Options on Spot U.S. Dollar Contracts

The reference premium for call and put options is calculated according to equations (2.1) and (2.2), respectively:

$$PRCALL_n = S \times e^{(-q_n T_n)} \times N(d_1) - K \times e^{(-r_n T_n)} \times N(d_2) \quad (2.1)$$

$$PRPUT_n = -S \times e^{(-q_n T_n)} \times N(-d_1) + K \times e^{(-r_n T_n)} \times N(-d_2) \quad (2.2)$$

where

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r_n - q_n + \frac{\sigma^2}{2}\right) T_n}{\sigma\sqrt{T_n}} \quad (2.3)$$

$$d_2 = \frac{\ln\left(\frac{S}{K}\right) + \left(r_n - q_n - \frac{\sigma^2}{2}\right) T_n}{\sigma\sqrt{T_n}} \quad (2.4)$$

S is the closing price of the option's underlying asset (see the B3 PRICING MANUAL – FUTURES CONTRACTS, section 2.1);

r_n is the exponential interest rate in continuous regime and on an annual basis related to the n contract month and calculated according to equation (2.5);

q_n is the foreign interest rate in exponential regime related to the currency that is the option's underlying asset. It is an exponential interest rate in continuous regime and on an annual basis related to the n contract month and calculated by equation (2.6);

T_n is the contract month term in calendar years pertaining to the marketplace in question, namely:

$$T_n = \frac{DU_n}{252}$$

where DU_n is the number of trading days between the calculation date and the expiration date of the n interpolated contract month n;

K is the option's strike price; and

σ is the option's volatility calculated according to section 2.2.

Calculation of the exponential interest rate

$$r_n = \ln(1 + TPre_{DI1}^{n}) \tag{2.3}$$

where:

$TPre_{DI1}^{n}$ is the fixed rate for the n contract month, calculated according to the exponential interpolation of the settlement prices of the One-Day Interbank Deposit Futures Contract (DI1) (see the B3 PRICING MANUAL – FUTURES CONTRACTS).

Calculation of the foreign exponential interest rate

$$q_n = \frac{252}{DU_n} \ln\left(1 + TPre_{DDI}^{n} \cdot \frac{DC_n}{360}\right) \tag{2.4}$$

where:

$TPre_{DDI}^{n}$ is the fixed rate for the n contract month, clean U.S. Dollar spread calculated according to the exponential interpolation of the settlement prices of U.S. Dollar spread futures contracts (see the B3 PRICING MANUAL FUTURES CONTRACTS, sections 1.2 and 1.3).

DU_n is the number of trading days between the calculation date and the expiration date of the n interpolated contract month;

DC_n is the number of calendar days between the calculation date and the expiration date of the n interpolated contract month;

Reference premium on the last trading day

The reference premium for call and put options is calculated according to equations (2.7) and (2.8), respectively:

$$PRCALL_n = Maximum[S - K; 0] \quad (2.5)$$

$$PRPUT_n = Maximum[K - S; 0] \quad (2.6)$$

where:

S is the Brazilian Reals to U.S. Dollar exchange rate, according to the PTAX800 sell rate published by the Central Bank of Brazil on the date corresponding to the last business day, or the business day before the option's expiration if the date is not a business day;

K is the option's strike price.

Reference premium on the expiration date

The reference premium for call and put options is calculated according to equations (2.7) and (2.8), respectively, considering the Brazilian Reals to U.S. Dollar exchange rate of the previous business day (according to the PTAX800 sell rate published by the Central Bank of Brazil on the date corresponding to the last business day, or on the business day before the expiration date.

2.2 Calculation of volatility for Option on Spot U.S. Dollar Contracts

The volatility for Options on Spot U.S. Dollar Contracts will be computed through the Stochastic Volatility Inspired (SVI) parameterization.

The formula for the SVI parameterization is:

$$var(x) = \sigma_{BS}^2 = a + b\left\{\rho(x-m) + \sqrt{(x-m)^2 + \sigma^2}\right\} \qquad (2.7)$$

where σBS is the implied volatility used in equations (2.1) and (2.2), $x = \ln(K/Fn)$, with K being the strike price and Fn, the future of the underlying asset in reference to the n contract month. For the U.S. Dollar, the future can be calculated as:

$$F = S\exp[(r_n - q_n)T_n]$$

The parameters of the equation (2.9) are estimated by minimizing the fitness function with the collected data (for details about the data collection see section 4):

$$f_{obj} = \sum_{i=1}^{N}(f_i - y_i)^2$$

where:

N is the volatility quantity obtained in the collected data;

f_i is the function of the model adopted in the adjustment, SVI parameterization formula;

y_i is the implied volatilities obtained in the collected data.

2.3 Volatility adjustment for Options on Spot U.S. Dollar

With Options on Spot U.S. Dollar, the settlement exchange rate is determined on the business day before the expiration date ($t-1$ PTAX), meaning that there is one day with no volatility when the contract's expiration date is considered.

Given that the expression for calculation of the reference premium used by the B3 considers the settlement date as a business day also, the volatility surface must be adjusted.

As the information that the brokerage houses submit to B3, which is used as an input for publication of the reference surface, does not consider the last volatility day, the adjustment executed in the volatility is calculated by:

$$\sigma^2_{Bolsa,i,j} = \sigma^2_{Broker,i,j} \cdot \frac{DU_j}{DU_j + 1}$$

where the indices i and j refer respectively to each Delta and to each contract month of the informant's surfaces (σ_{Broker}) and of that published by B3 (σ_{Bolsa}).

3 INTEREST RATES

3.1 Options on IDI

The reference premium for call and put options is calculated by equations (3.1) and (3.2), respectively:

$$PRCALL_n = e^{(-r_n T_n)} \times [S_n \times N(d_1) - K \times N(d_2)] \quad (3.1)$$

$$PRPUT_n = e^{(-r_n T_n)} \times [-S_n \times N(-d_1) + K \times N(-d_2)] \quad (3.2)$$

where:

$$d_1 = \frac{\ln\left(\frac{S_n}{K}\right) + \left(\frac{\sigma^2}{2}\right) T_n}{\sigma\sqrt{T_n}} \quad (3.3)$$

$$d_2 = \frac{\ln\left(\frac{S_n}{K}\right) - \left(\frac{\sigma^2}{2}\right) T_n}{\sigma\sqrt{T_n}} \quad (3.4)$$

r_n is the exponential interest rate in continuous regime and on an annual basis corresponding to the n contract month calculated according to equation (3.5);

T_n is the term in calendar years for the marketplace in question, that is:

$$T_n = \frac{DU_n}{252}$$

DU_n is the number of trading days between the calculation date and the expiration date of the n interpolated contract month;

S_n is the closing price of the option’s underlying asset, for this option the closing price is the forward IDI value calculated from the fixed rate for the n contract month (in reference to the option’s contract month) of the forward interest rate structure obtained from the One-Day Interbank Deposit Futures Contract (DI1).

$$S_n = IDI_0 \times e^{(r_n T_n)}$$

IDI_0 is the $IDIDI2009$ B3 index corresponds to the index corrected by the average rate of One-Day Interbank Deposits (DI), from the base date in 2009 until the calculation date;

K is the option's strike price;

σ is the option volatility calculated as set out in section 5.3.

Calculation of the exponential interest rate

$$r_n = \ln(1 + TPre^{n}_{DI1}) \quad (3.5)$$

where:

$TPre^{n}_{DI1}$ is the fixed rate for the n contract month, calculated by the exponential extrapolation of the settlement prices for the One-Day Interbank Deposit Futures Contract (DI1) (see the B3 PRICING MANUAL – FUTURES CONTRACTS).

3.2 Options on DI1 Futures

The reference premium for call and put options is calculated by equations (3.6) and (3.7), respectively:

$$PRCALL_n = \delta \times [S' \times N(d_1) - K' \times N(d_2)] \quad (3.6)$$

$$PRPUT_n = \delta \times [-S' \times N(-d_1) + K' \times N(-d_2)] \quad (3.7)$$

where:

$$d_1 = \frac{\ln\left(\frac{S'}{K'}\right) + \left(\frac{\sigma^2}{2}\right) T^{DC}_{C,n}}{\sigma\sqrt{T^{DC}_{C,n}}} \quad (3.8)$$

$$d_2 = \frac{\ln\left(\frac{S'}{K'}\right) - \left(\frac{\sigma^2}{2}\right) T^{DC}_{C,n}}{\sigma\sqrt{T^{DC}_{C,n}}} \quad (3.9)$$

$$K' = \left((1+K)^{T_{L,n}^{DU}-T_{C,n}^{DU}} - 1\right) \times \frac{1}{T_{L,n}^{DC} - T_{C,n}^{DC}}$$

$$S' = \left(\frac{PU_C}{PU_L} - 1\right) \times \frac{1}{T_{L,n}^{DC} - T_{C,n}^{DC}}$$

$$\delta = PU_L \times \frac{T_{L,n}^{DC} - T_{C,n}^{DC}}{1 + K'(T_{L,n}^{DC} - T_{C,n}^{DC})}$$

PU_L is the settlement price of the One-Day Interbank Deposit Futures Contract (DI1) with expiration of the option's underlying asset;

PU_C is the settlement price of the One-Day Interbank Deposit Futures Contract (DI1) with expiration of the option's underlying asset;

K is the option's strike price, exponential interest rate, in continuous regime and on an annual basis;

$T_{L,n}^{DC}$, $T_{C,n}^{DC}$ terms for PU_L and PU_C, respectively, in calendar years/days.

$T_{L,n}^{DU}$, $T_{C,n}^{DU}$ terms for PU_L and PU_C, respectively, in calendar years pertinent to the marketplace in question, namely:

$$T_{L,n}^{DU} = \frac{DU_{L,n}}{252} \text{ and } T_{C,n}^{DU} = \frac{DU_{C,n}}{252}$$

with $DU_{L,n}$ and $DU_{C,n}$ being the number of trading days between the calculation date and expiration date n;

σ is the option's volatility calculated as per section 3.4.

3.3 Calculation of volatility for Options on DI1 Futures

The volatility for options on DI1 futures will be computed through the SABR parameterization.

The formula used for the SABR parameterization is as follows:

$$\sigma_{BS}(S,K) = \frac{\alpha}{(SK)^{\frac{1-\beta}{2}}\left(1+\frac{(1-\beta)^2}{24}ln\left(\frac{S}{K}\right)^2+\frac{(1-\beta)^4}{1920}ln\left(\frac{S}{K}\right)^4\right)}\frac{z}{x(z)}$$

$$\times\left(1+\left(\frac{(1-\beta)^2}{24}\frac{\alpha^2}{(SK)^{1-\beta}}+\frac{1}{4}\frac{\rho\beta\upsilon\alpha}{(SK)^{\frac{1-\beta}{2}}}+\frac{2-3\rho^2}{24}\upsilon^2\right)(T)\right)$$

where:

σ_{BS} is the implied volatility of the Black-Scholes model;

K is the strike price;

S is the closing price of the option's underlying asset;

$$z = \frac{\upsilon}{\alpha}(SK)^{\frac{1-\beta}{2}}ln\left(\frac{S}{K}\right);$$

$$x(z) = ln\left(\frac{\left(\sqrt{1-2\rho z+z^2}\right)+z-\rho}{1-\rho}\right);$$

The parameters for the SABR equation are estimated by minimizing the fitness function with the collected data (for details about the collected data see section 4):

$$f_{obj} = \sum_{i=1}^{N}(f_i - y_i)^2$$

where:

N is the volatility quantity obtained in the data collection;

f_i is the function of the model adopted in the adjustment, SABR parameterization formula;

y_i is the implied volatilities obtained in the data collection.

3.5 COPOM Option Contracts

A set sequence of procedures is used to calculate the reference price for option premiums on COPOM Options contracts. If the first procedure cannot be applied, the second will be, and so forth successively until the settlement price is determined. The procedures involve the following definitions and conditions:

Electronic Closing Call: is a mechanism at the end of the trading session used to set a single price for all trades that occur at the closing call, even if orders have different prices.

Valid Trades are the trades in the option series that meet these conditions:

1. Present at the electronic closing call;
2. Quantity is greater than or equal to the minimum threshold set in the contract in question.

Valid Order is defined as an electronic closing call order that meets the following three conditions:

1. Present at the electronic closing call;
2. 30 seconds of minimum exposure;
3. Quantity is greater than or equal to the minimum threshold set in the contract/expiration in question.

If there is more than one valid bid/ask order, the highest bid order and the lowest ask order will be considered to be the valid order.

Valid offer spread. The difference between the price of the best valid bid price and the best valid ask price that is equal to or less than the limit established for the contract in question.

The procedure for determining the premium for each series is:

P1. The premium will be the price established at the electronic closing call for the expiration in question from valid trades.

P2. If procedure P1 cannot be applied, the premium for the series in question will be the difference between the average valid bid prices and valid ask prices with the valid bid-ask spreads for that contract.

P3. If procedure P2 cannot be applied, the premium for the series in question is given by the average of the day's trades weighted by the number of contracts, provided the total number of contracts exceeds the minimum needed for a valid premium in the Electronic Closing Call. The trades considered are the ones done till the end of the closing call.

P4. The theoretical model presented below gives the theoretical premium.

The theoretical model estimates the probability of each strike being the COPOM decision. By means of the probabilities the option premium is computed as

$$Premio_K = \frac{\mathrm{Prob_K}}{\left(1 + PRE^{DU}\right)^{DU/252}}$$

Where

K: option strike

Prob_K: probability of strike *K*

DU: business days to the option maturity

PRE^{DU}: risk neutral rate for maturity *DU*

It should be noted that when the implicit probabilities of the contract premiums defined by P1 and P2 add up to a total value above 1, these premiums will be maintained without undergoing adjustments. On the other hand, if the probabilities obtained by P3 or P4 do not total 1, they will be adjusted to add up to 1. This adjustment is made only on the probabilities obtained by P3 or P4, and is done by dividing each by the sum of the probabilities of all strikes defined by these procedures.

The premiums defined by P3 and P4 respect the valid offers of the closing call.

Theoretic Uniform Model

The probabilities will be determined by a uniform adjustment over the previous day's probabilities as described in the sequence. When there is no information on the previous day, it will be used a model based on DI1 futures.

$p_{i,k}$ denotes the strike probabilities with information from P1, P2, P3 or P4,

$p_{j,k}$ denotes the probabilities needing to be calculated,

$\bar{p}_{j,k}$ denotes the probabilities of the previous day’s strike prices needing to be calculated. Defining the uniformity factor as:

$$\alpha_k = \frac{1 - \sum p_{i,k}}{\sum \bar{p}_{j,k}}$$

the probabilities are defined as $p_{j,k} = \alpha_k \, \bar{p}_{j,k}$. In case of $\sum \bar{p}_{j,k} = 0$ it will be defined $\alpha_k = \frac{1 - \sum p_{i,k}}{number\ of\ probabilities\ \bar{p}_{j,k}}$.

Modelo teórico pelo futuro de DI1

When it is the very first day of a tenor or the calculation day is COPOM day or it is relevant economic indexes publication -relevant for the COPOM Options- such as inflation rate the model used will be one based on DI1 futures based on the hypothesis in the sequel.

Hypothesis 1: Only the COPOM causes changes on the CDI rate.

Forward rate step

The forward rate, $f_{j,}$ estimation for each tenor *j* of the copom option til the mautrity is computed

$$f_0 = CDI$$

$$f_j = \begin{cases} -1 + r_{j-1}^{\frac{252}{VF_j - V_j}} & se\ VF_j \neq V_j \\ \frac{rate_{VF_j}}{100} & se\ VF_j = V_j \end{cases} \quad j = 1, \ldots, tot$$

For $r_0 = \frac{\left(1+\frac{rate_{VF_1}}{100}\right)^{VF_1/252}}{(1+f_0)^{\frac{V_1}{252}}}$ and

$$r_j = \frac{\left(1 + \frac{rate_{VF_{j+1}}}{100}\right)^{VF_{j+1}/252}}{\prod_{i=0}^{j}(1 + f_i)^{\frac{V_{i+1} - V_i}{252}}} \quad j = 1, \ldots, tot - 1$$

where

V_j : number of business days minus one to maturity *j*.

VF_j : number of business days to the maturity of DI1 future immediately after the option maturity *j*.

$rate_{VF_j}$: interest rate implied in DI1 future of tenor VF_j .

tot: number of different tenors to the maturity is being computed (V_{tot} is the tenor which the model is being applied).

Jump step

The expected Copom decision is computed by the difference of the forwards rates $jump = f_{tot} - f_{tot-1}$.

Probabilities step

a) If $jump \leq K_0$ (minimum strike of maturity V_{tot}) then $Prob_{K_0} = 100$ and the other strikes get probability 0.
b) If $jump \geq K_N$ (maximum strike of maturity V_{tot}) then $Prob_{K_N} = 100$ and the other strikes get probability 0.
c) On the other hand, if $K_a \leq jump \leq K_p$, then

$$Prob_{K_a} = 100 * \frac{K_p - jump}{K_p - K_a}$$

and $Prob_{K_p} = 100 - \mathrm{Prob}_{\mathrm{K_a}}$

The other strikes get probability 0.

Prices step

For each strike,

$$Premio_K^{D+0} = \frac{\mathrm{Prob_K}}{\left(1+PRE_{D+0}^{V}\right)^{DU/252}}$$

4 COMMODITIES

4.1 Commodities options

The reference price for the call and put Options are given in equations (4.1) and (4.2), respectively:

$$PRCALL_n = e^{(-r_nT_n)} \times \left(F_n \times N(d_1) - K \times N(d_2)\right) \quad (4.1)$$

$$PRPUT_n = e^{(-r_nT_n)} \times \left(K \times N(-d_2) - F_n \times N(-d_1)\right) \quad (4.2)$$

where:

$$d_1 = \frac{\ln\left(\frac{F_n}{K}\right) + \left(\frac{{\sigma_n}^2}{2}\right)T_n}{\sigma_n \times \sqrt{T_n}} \quad (4.3)$$

$$d_2 = d_1 - \sigma_n \times \sqrt{T_n} \quad (4.4)$$

F_n = the closing price of the option underlying future contract;

r_n = the exponential interest rate in continuous regime and on an annual basis corresponding to the n contract month calculated according to equation (3.5);

T_n = the term in calendar years for the marketplace in question, that is: $T_n = \frac{DU_n}{252}$

DU_n= the number of trading days between the calculation date and the expiration date of the n interpolated contract month;

K = option strike price;

σ = volatility for the option.

The volatility surface comes from collecting data or, in the cases that there is an equivalent market with liquidity abroad, it is used the abroad volatility.

5 CRITERIA FOR COLLECTING IMPLIED VOLATILITY DATA FOR OPTIONS ON CURRENCY AND INTEREST RATES

Options on Spot U.S. Dollar, IDI and DI1 use collections of implied volatility surfaces submitted by brokerage houses that are part of the pool of informants (brokerages houses that are the most active in the market under assessment).

For the options on Spot U.S. Dollar and DI1, to ensure the quality of the information used in the construction of the volatility surface, the generation process filters the informants data by a non-arbitrage criterion and for outliers. In other words, the data does not meet this criterion are not considered when constructing the reference surface. These criteria are also used in the final validation of the reference delta volatility surface published by B3.

Once the informants' data are assessed and filtered, an adjustment is made to each *smile* following a parameterization that meets the aforementioned non-arbitrage criterion.

5.1 Non-arbitrage criteria for implied volatility surface

The main non-arbitrage criteria assessed for the implied volatility data for a call option are as follows:

I) A call option price decreases to the same proportion as the strike price:

$$\frac{\partial C}{\partial K} < 0$$

II) A call option price increases to the same proportion as the maturity:

$$\frac{\partial C}{\partial T} > 0$$

III) The convexity of the call option premium on the basis of the strike price must be positive:

$$\frac{\partial^2 C}{\partial K^2} \geq 0$$

IV) Finally, if extrapolation is required, the following conditions for $K \to 0$ and $K \to \infty$ apply, where X_0 is the value of the option's underlying asset on t_0:

$$\lim_{K \to 0} C = X_0$$
$$\lim_{K \to \infty} C = 0$$

To apply the criteria from I to III, strike prices are determined for each vertex surface of each informant.

5.2 Statistical criteria for implied volatility surface

On every trading day there is implied volatility surface data collection from different informants. As this is information about the premium of options traded on the same market, significant dispersal within it is not expected. Therefore, we assess each submitted data (σ_i) in relation to a confidence interval (I.C.) determined by the arithmetic mean ($\bar{\sigma}$) which is obtained by:

$$\bar{\sigma} - t(I.C., N-1)\frac{s}{\sqrt{N}} < \sigma_i < \bar{\sigma} + t(I.C., N-1)\frac{s}{\sqrt{N}}$$

where s is the standard deviation of the sample of informants and t determines the multiplicative factor taking into consideration the size of the sample via the t-student distribution, specifically using N as the number of informants for each smile.

5.3 Statistical criteria for option on IDI trading strategies

Due to the characteristics of the Options on IDI market, especially the fact that liquidity is concentrated into strategies (for example, call spread, put spread and butterfly), volatility calculations of these options might produce discrepancies between the prices of the strategies calculated based on data sample and traded prices.

Seeking to reduce these discrepancies there is a daily collection of the more traded options bid/ask, both strategies and individual options. These trades are priced with each collected volatility smile and the final volatility is defined by the combination of collected smiles that minimizes the quadratic distance between the informed price and the computed for the set of strategies evaluated.

Furthermore, while daily collections are carried out with strategy informants, the stock of strategies is also analyzed aimed at the quality of price formation for the strategies that may be open.

At the same time, as IDI option premiums are very sensitive to volatility interpolations by delta, for contract months with greater liquidity the implied volatility is extracted from premiums by strike prices communicated by the pool of informants. From this volatility structure, another volatility surface is obtained by standardized delta for each informant and the respective premiums of the traded strategies are calculated. This new smile also is considered to select the final volatility which minimizes the discrepancies as mentioned before. Volatility surface extraction from the premiums considers the call premiums for a delta above 50% and put premiums for the remainder, the strike prices between 0.5% and 99.95% deltas and strike prices with premiums above the intrinsic value.

6 UTILITIES FOR OPTIONS CALCULATIONS

To find the volatilities for each option series it is necessary to convert the volatility *smiles* in delta for volatility smiles in strike price and interpolate this curve in the strike price of each option series.

6.1 Conversion of Delta into Strike Price

For a call option we have $\Delta_F = N(d_1)$ and, therefore, the strike price formula from the *delta* is:

$$K = \exp\left[\frac{\sigma^2}{2}T - N^{-1}(\Delta_F)\sigma\sqrt{T}\right] \cdot A$$

Where N^{-1} is the inverse of the normal cumulative function and A is a function of S_0, r_1 and r_2 and is defined in accordance with the option's underlying asset, as follows:

- Options on Spot U.S. Dollar: A is the settlement price of the U.S. Dollar Futures Contract with the same contract month as the option, if the option's contract month coincides with the contract month of a futures contract. If there is no futures contract with the same contract month as the option, A is the value obtained by formula (2.1) of the B3 Pricing Manual – Futures Contracts;
- IBOVESPA Index options: A is the settlement price of the index futures contract with the same contract month as the option;
- Options on IDI: A is the value of the IDI-09 spot economic index comprised of the fixed interest rate of the DI1 contract curve over the term until option's expiration;
- Options on DI FRA: A is the value of the FRA forward interest rate, which is the option's underlying asset.

6.2 Interpolation of the volatility smile

The interpolation models used to obtain the volatility of each strike price authorized for trading are the monotonic exponential and cubic splines. When the strikes get a volatility with delta lower 1% will be used de same volatility as the delta 1% for that strikes, similarly the strikes with delta higher than 99% will get the volatility of delta 99%.

The exponential interpolation formula is:

$$\sigma_i = \sigma_a \cdot \left(\frac{\sigma_p}{\sigma_a}\right)^{\frac{K_i - K_a}{K_p - K_a}}$$

and the cubic spline interpolation model is:

$$\sigma_i = \sigma_a \cdot h_{00}(K_i^*) + \left(K_p - K_a\right) \cdot m_a \cdot h_{10}(K_i^*) + \sigma_p \cdot h_{01}(K_i^*)$$

$$+(K_p - K_a) \cdot m_p \cdot h_{11}(K_i^*)$$

where σ_i is the volatility the i option series and K_i is the series strike price. K_p and K_a are vertices of the volatility smile curve in strike price and represent the strike prices before and after the K_i strike price. σ_p and σ_a are the reference volatilities for vertices K_p, K_a, $K_i^* = \frac{K_i - K_a}{K_p - K_a}$ and

- $h_{00}(x) = (1 + 2x)(1 - x)^2$;
- $h_{10}(x) = x(1 - x)^2$;
- $h_{01}(x) = (3 - 2x)x^2$;
- $h_{11}(x) = (x - 1)x^2$.

To define the m_a and m_p tangents, the following steps are applied in the sequence denoting the number of points with information $i = 1, \dots, n$.

1. Calculate $d_i = \frac{\sigma_{i+1} - \sigma_i}{K_{i+1} - K_i}$ for $i = 1, \dots, n - 1$.

2. Define $m_1 = d_1$, $m_n = d_{n-1}$ and $m_i = \frac{d_{i-1} + d_i}{2}$ if the $d_{i-1} and\ d_i$ sign is the same and if either are null and $m_i = 0$ if the sign is different or a part is null fora $i = 2, \dots, n - 1$.

3. Apply this step for $m_i \neq 0$. Define $\alpha_i = \frac{m_i}{d_i}$ and $\beta_i = \frac{m_{i+1}}{d_i}$ and evaluate the flat behavior If any of the following conditions have not been met:

   a. $\alpha_i + \beta_i - 2 \leq 0$;

b. $\alpha_i + \beta_i - 2 > 0 \; and \; 2\alpha_i + \beta_i - 3 \leq 0;$

c. $\alpha_i + \beta_i - 2 > 0 \; and \; \alpha_i + 2\beta_i - 3 \leq 0;$

d. $\alpha_i - \frac{1}{3}\frac{(2\alpha_i+\beta_i-3)^2}{(\alpha_i+\beta_i-2)} \geq 0$

then redefine $m_i = \alpha_i d_i \frac{3}{\sqrt{\alpha_i{}^2+\beta_i{}^2}}$ and $m_{i+1} = \beta_i d_i \frac{3}{\sqrt{\alpha_i{}^2+\beta_i{}^2}}$.

6.3 Temporal interpolation in the absence of data during collection

When there is no volatility communicated for the T contract month, it will be obtained following one of the methods given in the sequence, depending on the available information.

6.3.1 The T maturity is situated between two contract months with data

In this case, a liner interpolation is applied to the variation to assure its growing behavior. For each T contract month and each Δ the total variance of the $\sigma(\Delta)$ volatility is calculated by the equation (5.1)

$$V_T(\Delta) = \sigma_T^2(\Delta)T \tag{5.1}$$

The interpolation is executed for each smile Δ as follows

$$\sigma_T(\Delta) = \sqrt{\left(V_{T_a}(\Delta) + \frac{V_{T_p}(\Delta) - V_{T_a}(\Delta)}{T_p - T_a}(T - T_a)\right)T^{-1}} \tag{5.2}$$

where

T: is the business days of the contract month to be calculated;

T_a is the business days of the contract month immediately prior to the contract month to be calculated;

T_p is the business days of the contract month immediately subsequent to the contract month to be calculated;

$\sigma_T(\Delta)$: is the volatility in the Δ for the T contract month.

For IDI options the spline interpolation described in previous section is used

$$\sigma_T(\Delta) = \sigma_a(\Delta) \cdot h_{00}(T^*) + \left(T_p - T_a\right) \cdot m_a \cdot h_{10}(T^*) + \sigma_p(\Delta) \cdot h_{01}(T^*)$$

$$+(T_p - T_a) \cdot m_p \cdot h_{11}(T^*)$$

Where $\sigma_T(\Delta)$ is the volatility for Δ at maturity T. T_p and T_a are the immediately subsequent and prior maturities of maturity T. $\sigma_p(\Delta)$ and $\sigma_a(\Delta)$ are the T_p and T_a volatilities, $T^* = \frac{T-T_a}{T_p-T_a}$ and the functions $h_{00}(x), h_{10}(x), h_{01}(x)$ and $h_{11}(x)$ are the ones defined at section 6.2.

To define the m_a and m_p tangents, the steps defined at section 6.2 are follows by using $d_i = \frac{\sigma_{i+1}(\Delta) - \sigma_i(\Delta)}{T_{i+1}-T_i}$ for $i = 1, \ldots, n-1$, denoting by $i = 1, \ldots, n$ the whole points of information available for the interpolation.

6.3.2 The T maturity is prior to the contract months with data

In this case, the volatility smile is obtained by a combination of the instantaneous volatility of the Garch model and the volatility that contains data from the collection. Firstly, the $\hat{\sigma}$ instantaneous volatility is estimated via the Garch model(1.1)

$$\hat{\sigma}^2(t+1) = \omega + \alpha r^2(t) + \beta \hat{\sigma}^2(t)$$

Where r is the log return of the option's underlying asset considering the closing price of the calculation day: for the U.S. Dollar options the clean spread will be used and for DI1 options the corresponding FRA rate will be used.

In the second step, the annualized $\hat{\sigma}\sqrt{252}$ and the one−day term are used in the linear interpolation (5.2) as data on the previous contract month to obtain $\sigma_T(50)$, nessa interpolação o vencimento posterior é o primeiro vencimento informado na coleta. Com essa volatilidade ATM é calculado o prêmio entre a volatilidade ATM estimada e a volatilidade ATM do primeiro vencimento informado

$$premium = \frac{\sigma_T(50)}{\sigma_{T_*}(50)}$$

where T_* is the first contract month communicated in the data collection.

In the third step, the smile premium is applied to the first contract month to obtain the complete volatility smile of the T contract month, namely, $\sigma_T(\Delta) = premium * \sigma_{T_*}(\Delta)$.

6.3.3 The T maturity is subsequent to contract months with data

In this case, equation (5.3) is used

$$\sigma_T(\Delta) = \sigma_{T_*}(\Delta) \times premium \qquad (5.3)$$

where

$\sigma_{T_*}(\Delta)$: is the volatility of the furthest contract month submitted by the informants.

$premium$: is the ratio between the Garch volatilities for T contract months (maturity to be extrapolated) and T_* (furthest maturity with data submitted by the informants).

$$premium = \frac{\sigma_T(50)}{\sigma_{T_*}(50)}$$

Where $\sigma_T(50) = \sqrt{252\, V(T)}$ is

$$V(T) = V_L + \frac{1 - \exp(-aT \cdot 252)}{aT \cdot 252}\left(\hat{\sigma}^2(t+1) - V_L\right) \qquad (5.4)$$

$$V_L = \frac{\omega}{1-\alpha-\beta}$$

$$a = \ln\frac{1}{\alpha+\beta}$$

with:

T is the term corresponding to the option's contract month in business days;

α, β and ω are the Garch model coefficients (1.1);

$\hat{\sigma}^2(t+1)$ is the instantaneous variance calculated according to the autoregressive volatility formula of the Garch model (1.1).

$r(t)$ is the last instant of the series of returns (calculated at the day's closing);

$\hat{\sigma}^2(t)$ is the autoregressive variance estimator obtained from the application of the formula over the series of returns and considering the sample variance as the variance at the origin $\hat{\sigma}^2(t-N-1)$, for a series of N length returns.

The extrapolations of sections 6.3.2, 6.3.3 and 6.3.4 is done on a sample of 3 years to estimate de Garch parameters. Those extrapolations do not apply to IDI volatility. For IDI options, it only will be authorized the maturities between smiles with informants.

6.3.4 No contract month communicated

In this case, the first step is to calculate the σ_T volatility according to the Garch model temporal structure (1.1) set out in the previous section, equation (5.4). There is also an estimation of asymmetry and kurtosis samples.

The second step consists of completing the smile, where the approach is similar to that used for equity options. In this case, different strike prices are defined encompassing all possible deltas. For these strike prices and using the σ_T volatility, obtained in the previous step, the premium is calculated via the Corrado & Su formula in section 1.2.1.1. Through these premiums, the implied volatility is calculated via the pricing formula of the option type in question and associated to the corresponding Δ.

The third step consists of adjusting a cubic spline to the data of the previous step to obtain the volatility in standardized deltas (1.1).

6.4 Treatment of *outliers*

The calculations that involve the use of a series of historical data pass through an outliers filter. This filter is both quantitative and qualitative. The quantitative filter adjusts a t-Student distribution to the data and identifies as outliers the returns that surpass the interval of a 99,6% confidence level. This level represents the tolerance of one extreme event during the year (level=1/252). The confidence interval is for negative and positive returns, then the outliers total mass is divided in two.
Specifically, outliers are returns outside of the interval

$$\left(t^{-1}\left(\frac{1/252}{2}\right) * \hat{\sigma} + \mu \, , t^{-1}\left(1 - \frac{1/252}{2}\right) * \hat{\sigma} + \mu\right)$$

where
$t^{-1}(\alpha)$ is the inverse of the t-Student distribution adjusted to the data.
$\hat{\sigma}$ is the sample standard deviation of the data.
μ is the sample mean of the data.
For a sample of three years of option asset log-returns.

At the same time, a qualitative analysis of the returns is carried out taking into consideration macroeconomic events and news that impact the market, with the aim of assessing if the return was originated from repricing the asset with limited impact on the volatility standard, a scenario in which the return is zeroed in the sample.

6.5 Premium decimals publication

The minimum price published for each group of assets.

Asset	Minimum price	Decimals
Dolar	0.001	3 digits
Ibovespa	0.01	0 digits except for the minimum price
COPOM	0	2 digits
Others	0.01	2 digits

Change log

Version	Item changed	Change	Reason	Date
1	NA	NA	NA	Dec 14, 2016
2	Addition to sections (items 1.2 and 1.22).	Differentiate the capture window from option data.	Complement the Manual	Sep 1, 2017
3	Addition to sections 2 to 5	Addition	Complement the Manual	Oct 23, 2017
4	Change to section 3.3	Change	Formula correction	Aug 6, 2018
5	Change to section 5	Change	Text change	Aug 31, 2018
6	Addition to section 5	Addition	Complement of the Manual	Nov 30, 2018
7	Addition to sections 5.3 and 5.4	Addition	New methodology	Dec 7, 2018
8	Change to sections 4.4 and 5.2	Addition	Complementary methodologies	Jul 1, 2019
9	Addition to sections 3.5	Addition	New product	Set 3, 2020
10	Introduction	Addition	Arbitrage disclaimer	Jun 15, 2022
	COPOM options	Change	New methodology	
	Section 4	Addition	Commodities options	
	Section 5	Addition	Methodology details	
	Section 6.5	Addition	Minimum price	
	Section 6.4, 6.3.4 and 6.2	Addition	Methodology details	

11	Section 3.4	Exclusion	P3 method of informantes	Aug 29, 2022
12	Section 5.3 Section 5.4	Exclusion Change to 5.3 and text modification	Data selection	Fev 1, 2023
13	Section 3.4	Addition	Details of calculus	Dez 20,2023
	Section 6.3	Addition	Spline interpolation between different maturities of IDI options	
	Section 6.5	Change	Section name	
14	Section 1	Change	Alteration to include the method of determining the carrying cost (or convenience yield) rate for Ibovespa Index options.	Dez 13,2024

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